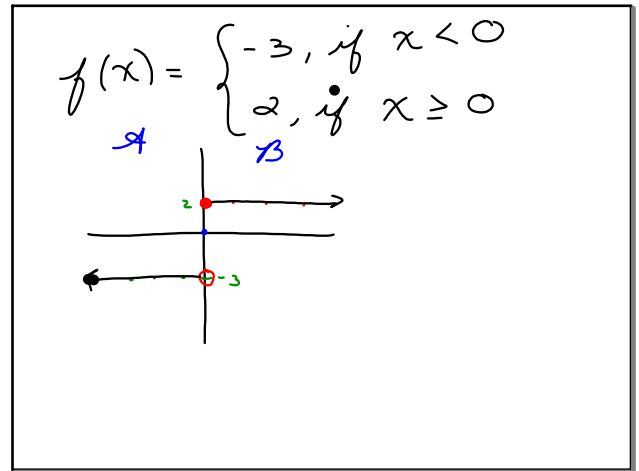


Jun 27-8:04 AM



Jun 27-8:26 AM

$$f(x) = 2x - 3$$

$$g(x) = 3x - 5$$

The Algebra of Functions

① + ② - ③ · ④ ÷

Combine ⑤

① + $(f+g)(x) = (2x-3) + (3x-5)$
 Domain: $(-\infty, \infty) = 2x-3+3x-5 = 5x-8$

② - $(f-g)(x) = (2x-3) - (3x-5)$
 Domain: $(-\infty, \infty) = 2x-3-3x+5 = -x+2$

③ · $(fg)(x) = (2x-3)(3x-5)$
 Domain: $(-\infty, \infty) = 6x^2 - 10x - 9x + 15 = 6x^2 - 19x + 15$

④ ÷ $(\frac{f}{g})(x) = \frac{2x-3}{3x-5}$
 Domain: $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$ $3x-5 \neq 0$ $3x \neq 5$ $x \neq \frac{5}{3}$

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⑤ Combine $(f \circ g)(x) = f(g(x))$

$$= f(3x-5)$$

$$= 2(3x-5) - 3$$

$$= 6x - 10 - 3$$

$$= 6x - 13$$

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$$f(x) = 2x + 4, \quad g(x) = x - 5, \quad h(x) = x^2$$

① $(f \circ g \circ h)(x) = f(g(h(x)))$

$$= f(g(x^2))$$

$$= f(x^2 - 5)$$

$$= 2(x^2 - 5) + 4$$

$$= 2x^2 - 10 + 4$$

$$= 2x^2 - 6$$

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find $(h \circ g \circ f)(x) =$

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$$(f \circ g)(x) = \sqrt{x^2 - 2}$$

① $f(x) = \sqrt{x}$

② $g(x) = x^2 - 2$

Trivial

$$f(x) = \sqrt{x^2 - 2}$$

$$g(x) = x$$

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$$(f \circ g)(x) = \frac{\sqrt{x}}{x-3}$$

① $x-3=0$
 $x=3$; so $x \neq 3$

② $x \geq 0$, so $[0, \infty)$

$[0, 3) \cup (3, \infty)$

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$$f(x) = 2x + 5$$

✓ Passes the Vertical Line test
* so, its a function

* But is it one-to-one?
• needs to pass the horizontal line test.

* If a function is one-to-one, then it has an inverse function.

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find the inverse of $f(x) = 2x + 5$

Steps

① change $f(x)$ to "y"
 $y = 2x + 5$

② Inturchange x & y
 $x = 2y + 5$

③ Solve $x = 2y + 5$ for y
 $\frac{x-5}{2} = \frac{2y}{2}$
 $\frac{x-5}{2} = y$
 $y = \frac{x-5}{2}$

④ Change "y" to $f^{-1}(x)$
 $f^{-1}(x) = \frac{x-5}{2}$ *Inverse notation*

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Test of Inverse

① $(f \circ f^{-1})(x) = f(f^{-1}(x))$
 $= f\left(\frac{x-5}{2}\right)$
 $= 2\left(\frac{x-5}{2}\right) + 5$
 $= x - 5 + 5$
 $= x$ ✓

② $(f^{-1} \circ f)(x) = f^{-1}(f(x))$
 $= f^{-1}(2x + 5)$
 $= \frac{(2x+5) - 5}{2}$
 $= \frac{2x}{2}$
 $= x$ ✓

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$$f(x) = x^2$$

So, f is a function but not one-to-one.
• It has no inverse!

$x \rightarrow g \rightarrow f \rightarrow y$

$f(x) = 2x + 5$	$f^{-1}(x) = \frac{x-5}{2}$
$x \mid f(x)$	$x \mid f^{-1}$
0 \mid 5	5 \mid 0
-1 \mid 7	7 \mid -1
-1 \mid 3	3 \mid -1

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